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Removal of Mixed Gaussian and Impulse Noise Using Data-driven Tight Frames

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Abstract

Image restoration is an important area in the field of image processing and analysis. This process is aimed at improving the quality of images with noise in image acquisition to restore the key and minor details of the original images. Impulse and Gaussian white noises are common types of noise. Traditional image restoration models cannot effectively remove impulse noise due to the strong randomness the latter. Therefore, impulse noise removal in images requires a considerable improvement. A variation model based on data-driven tight frames was proposed to recover the original image from the observed image with mixed Gaussian and impulse noise. First, the data-driven tight frame was imbedded in the variation model as a differential operator. Second, the exact solution of the variation model was obtained using augmented Lagrangian method-accelerated proximal gradient (ALM-APG) algorithm. Finally, the validity and practicability of the model and algorithm were verified through numerical experiments. Results demonstrate that (1) the ALM-APG algorithm has the advantages of low computational complexity and rapid convergence; (2) the variation model based on data-driven tight frames can effectively remove mixed Gaussian and impulse noise in images, and the peak signal-to-noise ratio (PSNR) value of the algorithm is higher than 30; (3) the proposed model demonstrates better image restoration effect than other image restoration models, and the PSNR has been increased by at least 10%. The proposed model can effectively remove the mixed Gaussian and impulse noise in images by providing a new method for removing such noise.

Keywords: Image restoration, Mixed Gaussian and impulse noise, Data-driven tight frames, ALM-APG algorithm

1. Introduction

Image processing has been a basic area in the field of computer vision with the development of artificial intelligence. In image acquisition, storage, or transmission, image pixels are likely to be polluted by various types of noise, such as Gaussian, impulse, Poisson, and mixed. Image analysis will be meaningless if noise is not eliminated in advance. Therefore, restoring an original image from the observed image has become the foundation of image processing. Additive white Gaussian and impulse noises are the most common types of image noise. Impulse noise has two categories, namely, salt-and-pepper and random-valued impulse noises, which are generally caused by data loss, memory defects, and deleted image transmission. For the salt-and-pepper noise in grayscale images, each destroyed pixel is replaced with 0 or 255 at a certain probability. For the random noise in a grayscale image, each destroyed pixel is replaced by a random value in $\{0,\cdots,255\}$.

The studies of removing additive Gaussian white and impulse noises have been conducted extensively, and numerous mature models and algorithms have been developed. However, removing mixed Gaussian and impulse noise should be further studied. The removal of mixed Gaussian and impulse noise is challenging given different properties and characteristics. Scholars have attempted to remove the mixed Gaussian and impulse noise in gray images by using variation, topological geometry, and combination models [1-6]. However, the peak signal-to-noise ratio (PSNR) value of image restoration is generally lower than 30. This result shows that the existing image restoration methods cannot adapt to the randomness of mixed Gaussian and impulse noise. Therefore, establishing an image restoration model on the basis of the characteristics of noise data is necessary to overcome the randomness of noise and effectively remove mixed Gaussian and impulse noise. An image restoration model, which can remove the mixed Gaussian and impulse noise, combined with the data-driven tight frame was proposed. This model is expected to improve the image restoration capability of mixed Gaussian and impulse noise and provide a reference for image restoration in engineering application.

2. State of the Art

Scholars have investigated the removal of mixed Gaussian and impulse noise in images. The algorithms of removing mixed Gaussian and impulse noise can be divided into three categories, namely, pixel domain- [7-14], regularization- [15-20], and patch-based algorithms [21-23]. Pixel domain-based algorithms generally remove mixed Gaussian and impulse noise through spatial nonlinear filtering and/or probabilistic/statistical technology. Regularization-based

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algorithms consider denoising an optimization. The regular term of image processing can be a total variation model or L_1 norm of wavelet frame coefficient. Patch-based algorithms realize image processing through a similarity with patch, therefore leading to high computational complexity and computation cost. In addition, the three algorithm categories are based on a variation model, which is the most extensively used model in image restoration. Rudin-Osher-Fatemi (ROF), which is a common variation model, can effectively restore images with segment time-like binary images (text and bar codes). Other categories of variation models based on the ROF model have been derived [15-23]. The variation model [22-27] based on tight wavelet frame has been successfully applied to image restoration. Study results show that the variation model based on tight wavelet frame has better effects than other variation models, such as the ROF, given the multi-resolution structure and redundancy of wavelet frames. On the basis of damaged information in images, Yan et al. designed an image restoration method by using blind information and successfully removed mixed Gaussian and impulse noise in images. However, this method was not extensively used given its high computational complexity [6]. Gong et al. developed a variation model based on tight wavelet frame and applied this model to image restoration with unknown noise. The processing effect for mixed Gaussian and impulse noise was unsatisfactory because a certain type of fixed noise was disregarded [27]. Shen et al. presented an image restoration method based on tight wavelet frame and tensor product and applied this model to gray image restoration with mixed Gaussian and impulse noise. However, the denoising effect still requires improvement because the method was not adaptable to image data [21]. Dong et al. constructed a 3D tight wavelet frame for 3D image restoration. The frame was unsuitable for 2D image restoration considering the particularity of a 3D tight wavelet frame [15]. Yang et al. used an image restoration method based on a variation model to remove the mixed Gaussian and impulse noise in 3D images and applied the method to image restoration with general noise. However, this method depends on the topological structure of images, and the denoising capability was low for general image data [23]. Wang et al. designed a tight wavelet frame for image restoration and constructed an image restoration model to remove Poisson and mixed Poisson-Gaussian noises. The processing capability for other types of noise is weak, although this method has effectively removed Poisson and mixed noises [22]. In addition, Cai et al. [25-26] established a correlation between wavelet frame and variation model, thereby establishing a theoretical basis for the superiority of variation model based on wavelet framework to several other variation models. Specifically, the variation model based on wavelet framework can adaptively select differential operators in various regions of a given image in accordance with the singularity order of potential solutions. Owing to the idea of tight wavelet frame, Cai et al. [24] constructed a data-driven tight frame based on the features of an image data structure. This frame can reconstruct images more accurately than the previous models.

On the basis of the above analysis, the processing capability of the image restoration methods for a certain type of noise is weak, although these methods, which remove the general unknown noise in images, can address many types of noise. Moreover, several shortcomings, such as high computation cost and dependence on image data, have been observed. The processing capability for other types of noise is weak and can hardly be extended to the studies of removing unknown noises, although the image restoration method for removing a fixed type of noise in images can significantly remove a specific noise. On the basis of the above analysis, to improve the denoising effect of the image restoration method and reduce the computation cost, a variation model based on data-driven tight frames was established to remove the mixed Gaussian and impulse noise in grayscale images. The fitting item of the variation model is composed of L_1-L_2 norm term, and the smooth terms consist of the L_1 norm items that contain data-driven tight frames. The exact solution of the variational model was obtained through augmented Lagrangian method-accelerated proximal gradient (ALM-APG) algorithm. The proposed model and algorithm were verified through numerical experiments and extended to the study of removing general unknown noises. This study is expected to provide a reference for image restoration in engineering application.

The remainder of this study is organized as follows. Section 3 describes the variation model and data-driven tight frames and discusses the process of constructing the image restoration variation model based on data-driven tight frames and the ALM-APG algorithm for solving the model. Section 4 verifies and analyzes the image restoration effect of the model and algorithm through numerical experiments. Section 5 summarizes the conclusions.

3. Methodology

3.1 Mathematical representation of mixed Gaussian and impulse noise

Mathematically, an image can be regarded as a column vector $x=\left(x_1,\cdots,x_d\right)^T$, where d is the total number of pixels in the images by arranging the columns. The range of pixels in a grayscale image is generally assumed to be [0,255]. The observed image y contaminated by mixed Gaussian and impulse noise is expressed as follows:

$$y_j=\begin{cases}x_j+n_j, & j\in\Omega\\ z_j, & j\in\Omega^c:=\left\{1,\ldots,d\right\}\setminus\Omega\end{cases}\tag{1}$$

where the value of $\Omega\in\left\{1,\cdots,d\right\}$ is unknown and called the observation field, $n_j, j\in\Omega$ is an independent identically distributed zero-mean additive Gaussian white noise, and $z_j, j\in\Omega$ is an independent identically distributed impulse noise (salt-and-pepper or random impulse noise). For the salt-and-pepper impulse noise, the noise pixel $z_j\in\left\{0,\cdots,255\right\}$ is valued at 0 or 255 with equal probability. For the random impulse noise, z_j is randomly valued at $\left\{0,\cdots,255\right\}$ with equal probability. The image domain Ω^c contaminated by impulse noise is assumed to be unknown, and each element of Ω^c is obtained using the entire image domain $\left\{1,\cdots,d\right\}$ through a Bernoulli experiment with a selected given probability $0\le p\le 1$. The value of Ω is assumed to be unknown. In the present study, a variation model based on data-driven tight frames is proposed to find the approximate solution of the original image x from the observed image.

3.2 Variation model theory

Image restoration is generally considered an inverse problem. An observed image b that contains noise is usually the sum of the original unknown image u and noise ε and is expressed as follows:

$$b = u + \varepsilon \tag{2}$$

The most commonly used method for restoring the original unknown image u is to solve the following variation model:

$$\min_u R_1(u) + R_2(u) \tag{3}$$

where $R_1(u)$ is the fitting term, which is used to approximate the noisy image, and $R_2(u)$ is the smoothing term used to maintain the image details, such as boundary and lines.

The smoothing term is generally determined by a priori assumption imposed on a potential solution. These assumptions are generally used in the sparsity of potential solutions in certain transform domains. These transform domains include gradient and wavelet. The sparsity of a tight wavelet frame transform domain is used as the a priori hypothesis of potential solutions. Therefore, $R_2(u) = \|Wu\|_1$, where W is the tight wavelet frame.

The fitting term $R_1(u)$ depends on a specified noise distribution. For example, $R_1(u)$ is expressed as follows when the noise is an additive Gaussian white noise:

$$R_1(u) = \|u - b\|_2^2 \tag{4}$$

$R_1(u)$ is expressed as follows when the image is contaminated by impulse noise:

$$R_1(u) = \|u - b\|_1 \tag{5}$$

However, the data-fitting term designed by the given noise may be ineffective on the mixed noise because the noise in the observed image rarely originates from a single distribution. A simple model was proposed to effectively remove the mixed Gaussian and impulse noise. The model expression is as follows:

$$\min_u \lambda_1 \|u - b\|_1 + \frac{\lambda_2}{2}\|u - b\|_2^2 + \rho\|Wu\|_1 \tag{6}$$

where, λ_1, λ_2, and ρ are the non-negative parameters used to balance the fitting and smoothing terms.

A new algorithm, namely, ALM-APG, is proposed to solve the model. This new algorithm is described in detail in Section 3. The numerical results show that the proposed model can effectively remove the mixed Gaussian and impulse noise combined with the proposed numerical algorithm, although Model (6) seems to be simple.

3.3 Data-driven tight frames

Data-driven tight frames approximate the input image accurately in accordance with the structural characteristics of the input image. Cai et al. [24] applied the data-driven tight frames to image restoration. The design process of the data-driven tight frame is as follows:

Data-driven tight frames

Input: image g (contaminated or uncontaminated)

Output: A discrete compact frame W^T constructed by a filter $\left\{a_i^{(K)}\right\}_{i=1}^{r^2}$

Main program

(I) Initialize tight frame filter $\{a_i^{(0)}\}_{i=1}^{r^2}$;

(II) for $k = 0,1,\cdots,K-1$ do

(1) Define $W^{(k)}$ in accordance with $\{a_i^{(k)}\}_{i=1}^{r^2}$;

(2) Let $\upsilon^{(k)} = T_\lambda(W^{(k)}g)$ and T_λ be the hard threshold operator;

(3) Construct matrix V, G in accordance with

$$\begin{cases} V = (\vec{\upsilon}_1, \vec{\upsilon}_2, \ldots, \vec{\upsilon}_N) \\ G = (\vec{g}_1, \vec{g}_2, \ldots, \vec{g}_N) \\ A = (\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_{r^2}) \end{cases};$$

(4) Perform SVD decomposition for VG^T and $VG^T = UDX^T$;

(5) $a_i^{(k+1)}$ represents the ith column of the matrix $A^{(k+1)} = 1/r\, XU^T$, $i = 1,\ldots,r^2$;

(III) Output $\{a_i^{(K)}\}_{i=1}^{r^2}$.

A set of low-pass and high-pass filters can be obtained using data-driven tight frames. A tight frame decomposition operator and a reconstruction operator can be established. Cai et al. [24] proposed the theory of this part. The specific theory is not elaborated here.

3.4 ALM-APG algorithm

The model cannot be directly solved by traditional methods because Model (6) is a least square problem with L_1 fitting and smoothing terms. To this end, the ALM is used to solve Model (6), and the inner subproblem is solved by using the APG [27].

First, Model (6) is transformed into the following problem:

$$\min_u \frac{\lambda_2}{2}\|u - b\|_2^2 + \beta^T|Au - c| \tag{7}$$

where $A = \begin{pmatrix} H \\ W \end{pmatrix}$, $c = \begin{pmatrix} b \\ 0 \end{pmatrix}$, and $\beta = \begin{pmatrix} \lambda_1 e \\ \rho\hat{e} \end{pmatrix}$. e and $\hat{e}$

are the vectors with an element of 1.

Second, Model (7) is transformed into the following equivalence problem by introducing a new variable z:

$$\min_{u,z} \ f(u,z) := \frac{\lambda_2}{2}\|u-b\|_2^2 + \beta^T|z| \quad s.t.\ Au + c = z \tag{8}$$

The ALM has been extensively used to solve convex programming problems. The augmented Lagrangian function of the original problem (8) associated with a given parameter $\sigma > 0$ is defined as follows:

$$\mathcal{L}_\sigma(u,z;y) = f(u,z) + \langle y, c - Au - z\rangle + \frac{\sigma}{2}\|c - Au - z\|^2 \tag{9}$$

The ALM iteration is used to solve the following inner subproblem. The following equation is obtained for a given $y^k, \sigma_k > 0$:

$$\left(u^{k+1}, z^{k+1}\right) \approx \arg\min_{u,z} \mathcal{L}_{\sigma_k}\left(u,z;y^k\right) \tag{10}$$

where

$$\mathcal{L}_{\sigma_k}\left(u,z;y^k\right) = f(u,z) + \frac{\sigma_k}{2}\left\|c - Au - z + \frac{1}{\sigma_k}y^k\right\|^2 - \frac{1}{2\sigma_k}\left\|y^k\right\|^2$$
$$= \frac{\lambda_2}{2}\|u-b\|_2^2 + \beta^T|z| + \frac{\sigma_k}{2}\left\|c - Au - z + \frac{1}{\sigma_k}y^k\right\|^2 - \frac{1}{2\sigma_k}\left\|y^k\right\|^2$$

First, z is considered to be minimized as follows:

$$\min_z \beta^T|z| + \frac{\sigma_k}{2}\left\|c - Au - z + \frac{1}{\sigma_k}y^k\right\|^2 = \frac{1}{\sigma_k}\sum_{i=1}^{m}\varphi_{\beta_i}(\eta_i) \tag{11}$$

where $\eta = \sigma_k(c - Au) + y^k$. $\varphi_\varepsilon(t)$ is the Huber function. It is defined as follows:

$$\varphi_\varepsilon(t) = \begin{cases} \frac{1}{2}t^2 & |t| \le \varepsilon \\ \varepsilon|t| - \frac{1}{2}\varepsilon^2 & |t| > \varepsilon \end{cases} \tag{12}$$

The optimal solution of (11) is defined by the following formula:

$$z = \frac{1}{\sigma_k}S_\beta(\eta) \tag{13}$$

For a given non-negative vector v, soft threshold operator S_v is expressed as follows:

$$S_v(x) = \mathrm{sgn}(x) \circ \max\{|x| - v, 0\} \tag{14}$$

where $\circ$ represents that the elements of the two vectors are the inner product, namely, $(x \circ y)_i = x_i y_i$. $\mathrm{sgn}(\cdot)$ is the symbol function. $\mathrm{sgn}(t)$ is the symbol of t, $\mathrm{sgn}(0) = 0$ when $t \ne 0$.

Second, the APG algorithm is used to calculate the optimal value u, to solve the following problem:

$$\min_u h(u) := \frac{1}{\sigma_k}\sum_{i=1}^{m}\varphi_{\beta_i}(\eta_i) + \frac{\lambda_2}{2}\|u-b\|_2^2 \tag{15}$$

The gradient of h is expressed by the following formula:

$$\nabla h(u) = -A^T\left(\mathrm{sgn}(\eta) \circ \min\{|\eta|, \beta\}\right) + \lambda_2(u-b) \tag{16}$$

where A column is full rank, and $h(x)$ is the strict convex function. Therefore, Minimization Problem (15) has a unique solution.

On the basis of the ALM and APG algorithms, the ALM-APG algorithm is used to solve the original problem, and the flowchart is summarized as follows:

ALM-APG algorithm

Initial: $\varepsilon > 0, u^0 = b, y^0 = y^{-1} = \mathbf{0}, L = 1, \sigma_0 = 1, k = 0$

while $\left\|\left(y^k - y^{k-1}\right)/\sigma_k\right\| > \varepsilon$ or $k = 0$ **do**

$u_0 = u_1 = u^k, t_0 = t_1 = 1$

for $i = 1$ *to* p **do**

$\bar{u}_i = u_i + \frac{t_{i-1} - 1}{t_i}\left(u_i - u_{i-1}\right)$

$u_{i+1} = \bar{u}_i - L^{-1}\nabla h(\bar{u}_i)$

$t_{i+1} = \frac{1 + \sqrt{1 + 4(t_i)^2}}{2}$

end for

$u^{k+1} = u_{p+1}, z^{k+1} = \frac{1}{\sigma_k}S_\beta\left(\sigma_k\left(c - Au^{k+1}\right) + y^k\right)$

$y^{k+1} = y^k + \sigma_k\left(c - Au^{k+1} - z^{k+1}\right)$

$\sigma_{k+1} = \sigma_k + 1$

$k = k + 1$

end while

4. Result Analysis and Discussion

Four groups of numerical experiments were conducted for the selected images, namely, “Lena.png,” “Peppers.png,” “Cameraman.png,” and “Flower.png,” to verify the effectiveness of the proposed model and algorithm for removing mixed Gaussian and impulse noise. The salt-and-pepper impulse noise was selected in the present study. The parameters of the mixed Gaussian and impulse noise are as follows: mean 0, variance 0.03, and impulse noise density 30%. To confirm the effect of the numerical experiment, the PSNR was used to evaluate the effect of image restoration. The expression of PSNR is as follows:

$$PSNR = -20\log_{10}\frac{\|u-\hat{u}\|_2}{N} \tag{17}$$

where u is the original image, $\hat{u}$ is the restored image, and N is the number of pixel points.

The denoising effects of the proposed image restoration algorithm are displayed in Fig. 1 and Table 1.

The images in Fig. 1 are Lena, Peppers, Cameraman, and Flower from top to bottom, and the original, noisy, and restored images are arranged from left to right. In Figure 1, the proposed image restoration model cannot only effectively remove the mixed Gaussian and impulse noise but also retain the image feature details, thereby indicating the favorable image restoration function of the proposed model. Data-driven tight frames continuously adjust the filter in the algorithm in accordance with the noise data and find the filter that is most suitable for the characteristics of the mixed Gaussian and impulse noise, to analyze the size and position of the noise. The variation model can accurately and effectively remove the mixed Gaussian and impulse noise in the image. The noise is found to be in the relatively lower frequency region than the detail features of the images. Therefore, the proposed image restoration model can not only remove the noise but also preserve the detailed features of the images.

Fig. 1. Restoration effect of noisy images

Table. 1. PSNR values

Image name	Lena	Pepper	Cameraman	Flower
PSNR value	30.97	30.78	31.09	30.56

In Table 1, the PSNR values of Lena, Peppers, Cameraman, and Flower are 30.97, 30.78, 31.09, and 30.56, respectively. If the PSNR value is greater than 30, then the effect of image restoration is favorable, thus indicating that the proposed image restoration model has an excellent denoising effect.

The proposed model was compared with the two methods that were recently published in SIAM Journal on Imaging Sciences and SIAM Journal on Multiscale Modeling and Simulation, the top journals in the field of image processing, respectively. The two methods are abbreviated as Yan [6] and Gong [27], respectively. The comparison results are presented in Fig. 2 and Table 2.

Fig. 2. Comparison of the image restoration effects

Table. 2. Comparison of the PSNR values

Methods	Lena	Pepper	Cameraman	Flower
Yan [6]	27.95	28.45	27.89	28.05
Gong [27]	28.44	28.85	29.08	28.85
Ours	30.97	30.78	31.09	30.56

Qualitatively, Fig. 2 illustrates that the proposed image restoration model can effectively remove the mixed Gaussian and impulse noise in the image and retain the detailed features of images, such as edges and angles, in comparison with the other two methods. From the quantitative analysis perspective, Table 2 lists that the PSNR values of the images processed through the proposed image restoration method are over 30, whereas the PSNR values of the two other methods are not greater than 30. Then, the proposed image restoration model has favorable image restoration effects and is, therefore, applicable to engineering or medical image background.

5. Conclusion

To effectively improve the image restoration effects for images with mixed Gaussian and impulse noise, a variation model based on data-driven tight frames was proposed to restore images with mixed Gaussian and impulse noise combined with data-driven tight frames and the variation model based on a patch. The validity of the model was verified through numerical experiments. The following conclusions could be drawn:

(1) The ALM-APG algorithm has low computational complexity and rapid convergence.
(2) Data-driven tight frames have high adaptability to image and noise data and can analyze the characteristics of noise. Therefore, the variation model based on data-driven tight frames can effectively remove the mixed Gaussian and impulse noise in images.
(3) The variation model based on data-driven tight frames has better image restoration capability than other image restoration methods. Moreover, the model depends on image data and has a strong adaptive capability.

The data-driven tight frame is combined with the variation model and applied to the field of image restoration. Numerical experiments show that the proposed model and algorithm can effectively realize image restoration in practice. The proposed model and algorithm can be further applied to other similar fields. However, several shortcomings in the study are observed. For example, the collected data of noise is deficient. The experimental data are all simulated data, thereby slightly limiting the practicability of the proposed model and algorithm. In future studies, the real noise data will be combined with the proposed model for further correction to describe its effectiveness of the proposed model in removing the measured noise in images.

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